

# ▲ PYRAMIDS ▲

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| (118) - 2 on L: | (121) - 1 on C: | (125) - 1 on L: |
| (119) - 1 on L: | (122) - 2 on R: | (126) - 2 on C: |
| (120) - 4 on C: | (123) - 1 on R: | (127) - 1 on C: |
|                 | (124) - 3 on C: |                 |

SEVEN discs have now been transferred. We have made 127 moves and have, as a matter of fact, completed just HALF of our task. Proceed—

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| (128) - 8 on R: | (166) - 2 on L: | (204) - 3 on R: |
| (129) - 1 on R: | (167) - 1 on L: | (205) - 1 on C: |
| (130) - 2 on L: | (168) - 4 on C: | (206) - 2 on R: |
| (131) - 1 on L: | (169) - 1 on C: | (207) - 1 on R: |
| (132) - 3 on R: | (170) - 2 on R: | (208) - 5 on C: |
| (133) - 1 on C: | (171) - 1 on R: | (209) - 1 on L: |
| (134) - 2 on R: | (172) - 3 on C: | (210) - 2 on C: |
| (135) - 1 on R: | (173) - 1 on L: | (211) - 1 on C: |
| (136) - 4 on L: | (174) - 2 on C: | (212) - 3 on L: |
| (137) - 1 on L: | (175) - 1 on C: | (213) - 1 on R: |
| (138) - 2 on C: | (176) - 5 on L: | (214) - 2 on L: |
| (139) - 1 on C: | (177) - 1 on R: | (215) - 1 on L: |
| (140) - 3 on L: | (178) - 2 on L: | (216) - 4 on C: |
| (141) - 1 on R: | (179) - 1 on L: | (217) - 1 on C: |
| (142) - 2 on L: | (180) - 3 on R: | (218) - 2 on R: |
| (143) - 1 on L: | (181) - 1 on C: | (219) - 1 on R: |
| (144) - 5 on R: | (182) - 2 on R: | (220) - 3 on C: |
| (145) - 1 on C: | (183) - 1 on R: | (221) - 1 on L: |
| (146) - 2 on R: | (184) - 4 on L: | (222) - 2 on C: |
| (147) - 1 on R: | (185) - 1 on L: | (223) - 1 on C: |
| (148) - 3 on C: | (186) - 2 on C: | (224) - 6 on R: |
| (149) - 1 on L: | (187) - 1 on C: | (225) - 1 on R: |
| (150) - 2 on C: | (188) - 3 on L: | (226) - 2 on L: |
| (151) - 1 on C: | (189) - 1 on R: | (227) - 1 on L: |
| (152) - 4 on R: | (190) - 2 on L: | (228) - 3 on R: |
| (153) - 1 on R: | (191) - 1 on L: | (229) - 1 on C: |
| (154) - 2 on L: | (192) - 7 on C: | (230) - 2 on R: |
| (155) - 1 on L: | (193) - 1 on C: | (231) - 1 on R: |
| (156) - 3 on R: | (194) - 2 on R: | (232) - 4 on L: |
| (157) - 1 on C: | (195) - 1 on R: | (233) - 1 on L: |
| (158) - 2 on R: | (196) - 3 on C: | (234) - 2 on C: |
| (159) - 1 on R: | (197) - 1 on L: | (235) - 1 on C: |
| (160) - 6 on L: | (198) - 2 on C: | (236) - 3 on L: |
| (161) - 1 on L: | (199) - 1 on C: | (237) - 1 on R: |
| (162) - 2 on C: | (200) - 4 on R: | (238) - 2 on L: |
| (163) - 1 on C: | (201) - 1 on R: |                 |
| (164) - 3 on L: | (202) - 2 on L: |                 |
| (165) - 1 on R: | (203) - 1 on L: |                 |

A Problem From The Ancients

# ▲ PYRAMIDS ▲

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| (239) - 1 on L: | (245) - 1 on L: | (250) - 2 on L: |
| (240) - 5 on R: | (246) - 2 on C: | (251) - 1 on L: |
| (241) - 1 on C: | (247) - 1 on C: | (252) - 3 on R: |
| (242) - 2 on R: | (248) - 4 on R: | (253) - 1 on C: |
| (243) - 1 on R: | (249) - 1 on R: | (254) - 2 on R: |
| (244) - 3 on C: |                 | (255) - 1 on R: |

And thus we have transferred, in 255 moves, 8 of the 64 discs which Brahma instructed his priests to transpose from one pyramid to another, with the admonition, "By the time you have executed this task, the end of the world will be near!"

As you may now realize, the 255 moves which you have just completed are but an infinitesimal part of those required to transpose the original 64 discs of this fascinating problem. Some idea of the endless task involved may be gained by comparison with the ancient "Chess-Board" problem. In that problem, one grain of corn was placed on the first square, 2 on the second, 4 on the third, and so on throughout the 64 squares. The total number of grains necessary for the last square amounts to 18,446,774,073,709,551,615! When we realize that the counting of a single billion at the rate of 100 a minute would take 19,024 years, the vast time allotted to the completion of the problem may be comprehended.

Yet the transposition of the 64 golden discs from one pyramid to the other would require an even greater period of time than the completion of the "Chess-Board" problem!

And so, when we transfer—in 255 moves—our 8 discs, we might indeed say that we have "touched upon infinity."

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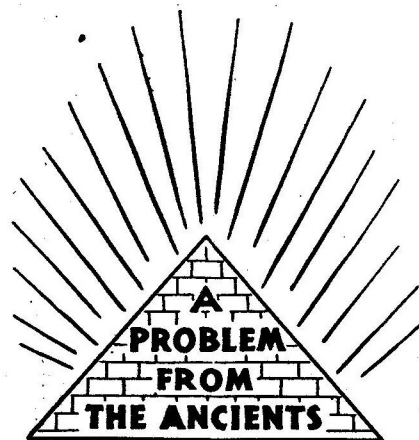
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FORM EX 100

A Problem From The Ancients

# The Solution of PYRAMIDS

CORRECTION NOTE  
MOVE NUMBER 106 SHOULD BE 2 ON L  
MOVES NUMBER 206-207-208 ARE DUPLICATED AS A GROUP  
ELIMINATE ONE OF THE GROUPS.



# ... and the prophet Brahma said unto his priests: "... By the time you

FROM time immemorial man has pondered over the mystery of the Pyramids of Egypt. The mathematical interpretation of the dimensions of these pyramids has led many learned students to believe that the ancient Egyptians had a deeper knowledge than exists today of the laws governing the vast network of our universe. It is known that far deeper meaning than the multiplication or division of numbers was discovered in mathematics by the ancient Egyptians. And the interpretation of these mathematical discoveries in relation to the workings of the Universe are believed to be embodied in the construction of the great pyramids which still rest on the sands of the Sahara Desert.

Our most learned students of the present day eagerly grasp and attempt to analyze even the slightest vestige of this mystery which has survived the ages.

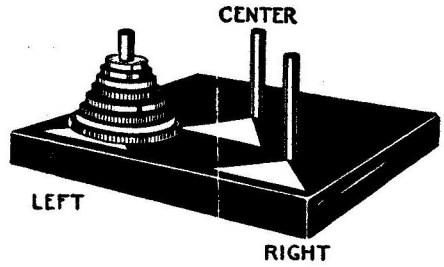
It is within the bounds of possibility that the problem which you are about to solve bears some relation to the mathematical interpretations evolved by the ancient Egyptians. It is a simplified version of a problem which has survived the ages and was conceived by the prophet Brahma who lived many centuries ago.

The original problem consisted of three pyramids arranged to form a triangle. Upon the left pyramid, 64 golden discs lay. The problem involves transferring these 64 discs from the left to the right pyramid—transposing one disc at a time only—and putting it either on a vacant pyramid or a larger disc. The vast labor involved in completing this transposition may be realized when we quote the words (in part) of Brahma to his priests: "...by the time you have executed this task, the end of the world will be near!"

By reducing the number of discs from 64 to 8 we have simplified the problem so that it can be solved in a very short time.

Although there are but eight discs to be transposed from the left pyramid to the right pyramid, 255 moves are necessary before the complete transposition of the eight discs is accomplished. This astounding total is reached due to the fact that after each disc has been transferred, the number of moves to transpose each succeeding disc is double the

total moves required to transpose the preceding disc. The ancient Egyptians obtained a complete transposition as follows:  
(For reference it will be desirable to distinguish the discs by numbers, the smallest one No. 1, the next larger 2, and so on, up to the largest which will be No. 8.)



The three pyramids we will assume to be placed in a row before us, distinguished by the letters—L, C, R, respectively, equivalent to LEFT, CENTER, RIGHT. The discs are arranged at the outset in proper order—No. 1 uppermost, and so on. We then proceed with the moves as follows:

- (1) - 1 on C:
- (2) - 2 on R:
- (3) - 1 on R:

We have now transferred two of the rings, and it will be observed that it has taken THREE moves to do it. One might conclude that it will only require eight times three moves to transfer all eight discs. Such is not the case, however, for before each succeeding disc can be brought to the base of the new heap, a constantly increasing number of transpositions will be necessary.

- We proceed:
- (4) - 3 on C:
  - (5) - 1 on L:
  - (6) - 2 on C:
  - (7) - 1 on C:

We have now transferred THREE of the discs.

- (8) - 4 on R:
- (9) - 1 on R:
- (10) - 2 on L:
- (11) - 1 on L:
- (12) - 3 on R:
- (13) - 1 on C:
- (14) - 2 on R:
- (15) - 1 on R:

FOUR discs are now transferred.

- (16) - 5 on C:
- (17) - 1 on L:
- (18) - 2 on C:
- (19) - 1 on C:
- (20) - 3 on L:
- (21) - 1 on R:
- (22) - 2 on L:
- (23) - 1 on L:
- (24) - 4 on C:
- (25) - 1 on C:
- (26) - 2 on R:
- (27) - 1 on R:
- (28) - 3 on C:
- (29) - 1 on L:
- (30) - 2 on C:
- (31) - 1 on C:

FIVE discs are now transferred, and it has taken 31 moves to accomplish this.

- (32) - 6 on R:
- (33) - 1 on R:
- (34) - 2 on L:
- (35) - 1 on L:
- (36) - 3 on R:
- (37) - 1 on C:
- (38) - 2 on R:
- (39) - 1 on R:
- (40) - 4 on L:
- (41) - 1 on L:
- (42) - 2 on C:
- (43) - 1 on C:
- (44) - 3 on L:
- (45) - 1 on R:
- (46) - 2 on L:
- (47) - 1 on L:
- (48) - 5 on R:
- (49) - 1 on C:
- (50) - 2 on R:
- (51) - 1 on R:
- (52) - 3 on C:
- (53) - 1 on L:
- (54) - 2 on C:
- (55) - 1 on C:
- (56) - 4 on R:
- (57) - 1 on R:
- (58) - 2 on L:
- (59) - 1 on L:
- (60) - 3 on R:
- (61) - 1 on C:
- (62) - 2 on R:
- (63) - 1 on R:

SIX discs are now transferred.

- (64) - 7 on C:
- (65) - 1 on L:
- (66) - 2 on C:
- (67) - 1 on C:
- (68) - 3 on L:
- (69) - 1 on R:
- (70) - 2 on L:
- (71) - 1 on L:
- (72) - 4 on C:
- (73) - 1 on C:
- (74) - 2 on R:
- (75) - 1 on R:
- (76) - 3 on C:
- (77) - 1 on L:
- (78) - 2 on C:
- (79) - 1 on C:
- (80) - 5 on L:
- (81) - 1 on R:
- (82) - 2 on L:
- (83) - 1 on L:
- (84) - 3 on R:
- (85) - 1 on C:
- (86) - 2 on R:
- (87) - 1 on R:
- (88) - 4 on L:
- (89) - 1 on L:
- (90) - 2 on C:
- (91) - 1 on C:
- (92) - 3 on L:
- (93) - 1 on R:
- (94) - 2 on L:
- (95) - 1 on L:
- (96) - 6 on C:
- (97) - 1 on C:
- (98) - 2 on R:
- (99) - 1 on R:
- (100) - 3 on C:
- (101) - 1 on L:
- (102) - 2 on C:
- (103) - 1 on C:
- (104) - 4 on R:
- (105) - 1 on R:
- (106) - 2 on R:
- (107) - 1 on L:
- (108) - 3 on R:
- (109) - 1 on C:
- (110) - 2 on R:
- (111) - 1 on R:
- (112) - 5 on C:
- (113) - 1 on L:
- (114) - 2 on C:
- (115) - 1 on C:
- (116) - 3 on L:
- (117) - 1 on R:

# have transferred these discs, the end of the World will be near - - - "!